On Hidden Variables—A Reply to Comments by Jauch and Piron and by Gunder

BOHM, D.
Birkbeck College, University of London, London, England
J. BUB
Department of Physical Chemistry, University of Minnesota, Minneapolis, Minnesota

Recently, Jauch and Piron\(^1\) have commented on two articles of ours on hidden variables.\(^2\) These comments have been extended by Gunder.\(^3\)

Jauch and Piron\(^1\) claim that if hidden variables existed, this would (as shown in their paper\(^4\)) lead to observable consequences which would contradict certain known facts of microsystems. On the other hand, in our paper we proposed a counterexample consisting of an explicitly formulated theory of hidden variables which was shown to be capable of fitting the known experimental facts underlying quantum mechanics to an arbitrarily high degree of accuracy. Thus our conclusions are in direct contradiction with those of Jauch and Piron.

In support of their point of view, Jauch and Piron say of their theory: “The validity of quantum mechanics is not assumed. Instead, one assumes only a lattice structure of yes-no experiments (called propositions) originating directly from experimental facts.”

With regard to this argument, we wish to emphasize that Jauch and Piron themselves admit that the above is an assumption. In the very same sentence, however, they also state that this assumption originates directly from experimental facts. Such a statement is ambiguous. Does it mean that the lattice structure itself is a fact? Or does it mean that the observed facts lead uniquely and inevitably to this lattice structure as the only possible set of theoretical ideas that could be logically compatible with the facts in question? Evidently, whichever way their statement is interpreted, it is wrong. For there are as yet no experiments in physics which show the lattice structure of propositions as a directly observed fact. At most, one can regard it as a possible axiom that may be compatible with the facts that are at present available. But then, as we have shown in our papers, one can propose a different set of axioms that are also compatible with these facts. Therefore the lattice structure of propositions cannot be a unique and inevitable inference from the known facts.

Further on Jauch and Piron object to our postulation of a nonlinear law for quantum mechanics. (Incidentally, we suggested that it applies universally and not merely—as stated by them—in a measurement process). They argue that it is contrary to good scientific methodology to modify a generally verified scientific theory for the sole purpose of accommodating hidden variables.

On the other hand, in our paper we suggested that, in this case, such a procedure is actually necessary for good scientific methodology. Unless one can criticize the basic postulates of a theory by showing what it would mean to contradict them, one will tend to be trapped in a closed cycle of thinking which suggests only the kinds of experimental questions that can be answered within the framework of the theory in question. However, to criticize the basic postulates of the quantum theory in this way it was necessary to see the falsity of von Neumann’s conclusion that these postulates are logically inevitable consequences of the observed facts. By showing that von Neumann’s conclusions are based on assumptions that go beyond what can actually be observed in experiment we opened the way to consider what it means to change these assumptions, and thus to the suggestion of new kinds of experiments that could, in principle, falsify the whole structure of the quantum theory as it is now generally formulated.\(^5\) On the other hand, if von Neumann’s arguments had been right, it would have been logically impossible for any experiment ever to be done that would not confirm the axiomatic structure of the quantum theory which he proposed.

The basic question at issue here is a point often overlooked: i.e., that the axioms of a theory stand on a different level from the experimental facts underlying the theory.\(^6\) It is therefore wrong to equate any set of axioms whatsoever with facts in the way that is done by Jauch and Piron (as well as by von Neumann). Rather, axioms are always assumptions from which one draws inferences about what is factually observable. If these inferences agree with the facts, the assumed structure of axioms is confirmed; and if they disagree, it is refuted. But if the axiomatic structure is confirmed by the facts available at a given time, this can never imply that no other axiomatic structure is possible that could agree with the same set of facts. Claims such as those of von Neumann and Jauch and Piron—that the facts necessitate a certain set of axioms—are therefore in contradiction with the very structure of the kind of reasoning that is involved in subsuming a specified body of scientific knowledge under a given axiomatic formulation.

As an example we can consider the axiomatizing of geometry. One knows from the available facts that parallel lines remain equidistant and do not meet. One set of axioms leading to inferences that agree with these facts are those of Euclid. However, as is well known, one can obtain equivalent inferences with regard to these facts if one adopts the axioms of elliptic geometry or hyperbolic geometry with the proviso that the departure of parallel lines from equidistance is too small to have been detected in existing types of experiments. Thus both sets of axioms lead to inferences that explain the known facts equally well. But they lead to different inferences when extended to a domain of distances whose properties are as yet untested and unknown. In addition, different geometrical postulates open the way

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\(^3\) Gunder, W., Physical Review, 134, A1388 (1964).
to explaining the possibility of new relationships between geometry and physical properties (such as gravitation).

An argument analogous to that of Jauch and Piron would then be the assertion that since the postulates of Euclid "originate directly from the experimental facts"—non-Euclidian geometries are impossible (as well as incompatible with good scientific methodology).

On the other hand, it is of course well known that the consideration of non-Euclidian geometries was both logically possible and physically useful even before factual observations were available that could distinguish between the two theories. Indeed, if physicists had believed that non-Euclidian geometries were excluded by the facts, they could never even have framed the question of how to study the nature of geometry experimentally. Similarly, if it is accepted that hidden variables are excluded by the facts, it becomes impossible to frame the question of how to decide experimentally whether or not there are hidden variables.

With regard to Gudder's paper, we wish to emphasize that its basic point of view does not disagree with ours (while being diametrically opposed to that of Jauch and Piron).

This is shown in his statement: "Thus, the author agrees with Bell, Bohm, and Bub to the extent that hidden variables cannot be excluded from quantum mechanics in an absolute sense, but only as far as certain models are concerned." We shall not, however, comment here on Gudder's detailed proposals for meeting some of our more technical criticisms of the work of Jauch and Piron. Nevertheless, we do wish to express agreement, at least in principle, with Gudder's suggestion that, by changing the model of Jauch and Piron, it may be possible to extend the class of theories that are not compatible with hidden variables (for example, beyond the class considered in von Neumann's model, based on the assumption of the linearity of observables).

As to Gudder's question of whether or not the additional complications introduced by hidden variables are justified, we have already commented on why we think that it is useful, at present, to explore the consequences of theories of this kind.

5. Experiments of this kind are already being carried out in a preliminary way in several places. See, for example, C. Papaliolios, Phys. Rev. Letters 18, 622 (1967).