Statistical Mechanics

August 26, 2015

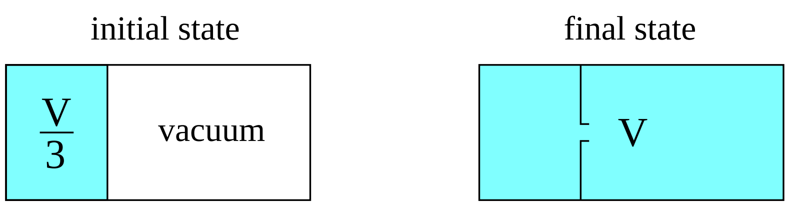
Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A classical monatomic *non-ideal* gas has the equation of state

$$\left(P + \frac{a}{v^2}\right)(v - b) = kT \quad , \quad \text{where} \quad v = \frac{V}{N}.$$

The gas is initially confined to 1/3 of the volume V of a container by a partition. The initial temperature of the gas is T_0. Then a hole is opened in the partition allowing the gas to expand into the rest of the container and attain a final volume V. The walls of the container are absolutely rigid and do not absorb or conduct heat. Find the final temperature of the gas after the expansion.

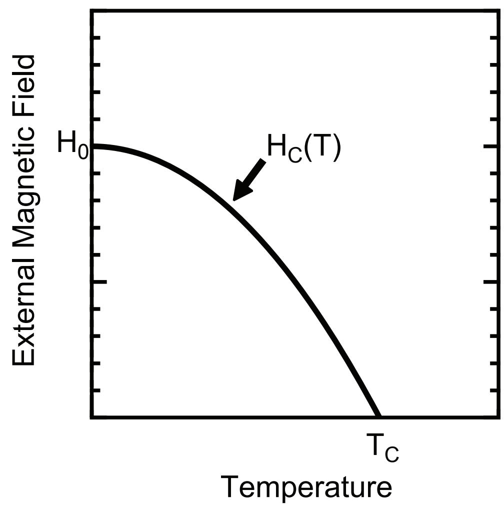


$$\text{Useful relations :} \quad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \quad , \quad C_V = \frac{3}{2}Nk$$

Problem 2

Consider a type I superconducting material with a parabolic coexistence curve separating the (uniform) superconducting and normal phases (see also figure below). Here, H is the external magnetic field and T is the temperature. Ignore the (tiny) magnetization of the normal phase. The critical field $H_C(T)$ separating the superconducting and the normal phase is given by:

$$H_C(T) = H_0 + aT + bT^3$$



Coexistence curve between superconducting and normal phase.

Questions:

a) Why must the coefficient a be zero?

b) Calculate the latent heat per unit volume as a function of T along the coexistence curve in terms of H_0 and T_C (a schematic is shown in the figure).

c) Calculate the discontinuity of the specific heat per unit volume at constant magnetic field along the coexistence curve.

Problem 3

Carbon Dioxide, CO_2, is a linear triatomic molecule with an electronic ground state of ${}^1\Sigma_g{}^+$. It possesses four normal modes of vibration; two bending modes and one in phase and one out of phase stretching modes ($f_1 = f_2 = 667.3\,\mathrm{cm}^{-1}$, $f_3 = 1383.3\,\mathrm{cm}^{-1}$, and $f_4 = 2439.3\,\mathrm{cm}^{-1}$). It also has a rotational constant $B = \hbar^2/(2I) = 0.390\,\mathrm{cm}^{-1}$ (where I denotes the moment of inertia). Assuming ideal behavior, evaluate the entropy and constant volume heat capacity at 298 K and 1 bar of pressure. Note that the molar mass for CO_2 is 44 grams per mole and the gas constant R has a value of 8.314 Joules per mole-Kelvin.