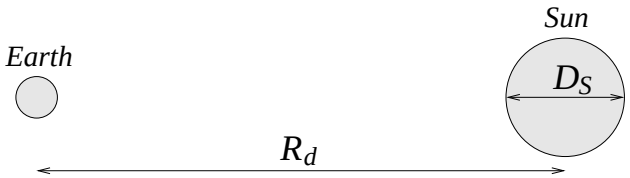
Statistical Mechanics

August 27, 2014

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

Estimate the average equilibrium temperature of the Earth's surface, T_E, under the black-body approximation, assuming that the Earth and the Sun are perfect 'black bodies'. The diameter of the Sun, D_S, is 1,390,000 km. The surface temperature of the Sun, T_S, is 5,778 K. The average distance of the Earth from the Sun, R_d, is 149,600,000 km.



Problem 2

Consider a quantum dot containing a single bound state with negative energy $-\epsilon$ It can be empty, occupied by one electron (spin-up or spin-down), or occupied by two electrons (both spin-up and spin-down). In the latter case, the Coulomb repulsion leads to an extra charging energy $U > 0$. The Hamiltonian thus reads

$$H = -\epsilon(n_1 + n_2) + U n_1 n_2$$

where $n_1,\ n_2 = 0, 1$ are the occupation numbers. The dot is embedded in a bulk semiconductor, which provides a constant chemical potential μ for the electrons.

a) Work in the grand-canonical ensemble and find the grand partition function of the quantum dot.

b) Find the average number of electrons on the dot as function of temperature and chemical potential.

c) Consider the case $\mu = 0$ and $U > \epsilon$. Discuss the behavior of the number of electrons on the dot for high and low temperatures.

d) What is the probability for double occupancy in the low-temperature limit?

Problem 3

Imagine a fictitious surface element in an ideal gas. Supposing that momentum transfer takes place through the surface element due to penetration by gas molecules, find the formula for calculating the pressure which both sides of the surface element exert upon each other. Assume that the gas molecules obey a Maxwellian velocity distribution.