Statistical Mechanics

August 28, 2013

1. Calculate the average magnitude of the momentum of a photon in a black-body radiation cavity at temperature T.

Useful equations:

$$\int_0^\infty \frac{x^{n-1}dx}{e^x - 1} = \Gamma(n)\zeta(n)\ ,\ \Gamma(n) = (n-1)!$$

$$\zeta(2) \approx 1.645\ ,\ \zeta(3) \approx 1.202\ ,\ \zeta(4) \approx 1.082.$$

Problem 2

An ideal classical gas, confined in a container with the linear size scale L, had been in thermal equilibrium at temperature T. Then a small hole of size $a << L$ was opened in the wall of the container for a short time interval t such that $a << v_0 t << L$ where v_0 is the r.m.s. velocity of the molecules in equilibrium. Find the r.m.s. velocity of the escaped molecules and compare it with v_0.

On the basis of the comparison, what would be the most immediate observable effect of the gas emission?

Problem 3

Atoms in a solid vibrate about their respective equilibrium positions with small amplitudes. Debye approximated the normal vibrations with the elastic vibrations of an isotropic continuous body and assumed that the number of vibrational modes $g(\omega)d\omega$ having angular frequencies between ω and $\omega + d\omega$ is given by

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\begin{aligned}
g(\omega) &= \frac{V}{2\pi^2}\left(\frac{1}{c_l^3} + \frac{2}{c_t^3}\right)\omega^2 & (\omega < \omega_D) \\
&= 0 & (\omega > \omega_D)
\end{aligned}
$$

Here c_l and c_t denote the velocities of longitudinal and transverse waves, respectively whilst ω_D is the Debye frequency. The Debye frequency is given by

$$\int_0^{\omega_D} g(\omega)d\omega = 3N$$

where N is the number of atoms and hence $3N$ is the number of degrees of freedom. Give an expression for the specific heat of a solid at constant volume with this model. Examine its temperature dependence at high as well as at low temperatures.

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