Electromagnetism

August 26, 2013

Work 4 (and only 4) of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A plane electromagnetic wave of frequency ω is normally incident from vacuum on a flat surface of a nonmagnetic material with electrical conductivity σ. Estimate the reflection coefficient of the incident wave in the limit $\omega << \sigma/\epsilon_0$.

Problem 2

A parallel-plate capacitor was lowered into water in a horizontal position, with water filling up the gap between the plates d=1.0 mm wide. Then a constant voltage V=500 V was applied to the capacitor. The process was carried out under normal conditions, i.e., at the density of water molecules present in the tank, the dielectric constant of the water is $\epsilon = 80\epsilon_0$. Find the water pressure increment in the gap.

Problem 3

From the surface of a wire of circular cross-section of radius a carrying a direct current I an electron escapes with a velocity v_0 perpendicular to the surface. Find what will be the maximum distance of the electron from the axis of the wire before it turns back due to the action of the magnetic field generated by the current.

Problem 4

A classical electromagnetic wave satisfies:

$$\vec{E} \cdot \vec{B} = 0 \quad \text{and} \quad \vec{E} \cdot \vec{E} = \vec{B} \cdot \vec{B}$$

Show that if this is correct in one reference frame it is correct in any reference frame (E and B are the electric and magnetic fields and Gaussian units are used).
Hint: Lorentz transformation of the electromagnetic fields E and B:

$$\vec{E}' = \gamma(\vec{E} + \vec{\beta} \times \vec{B}) - \frac{\gamma^2}{\gamma + 1}\vec{\beta}(\vec{\beta} \cdot \vec{E})$$

$$\vec{B}' = \gamma(\vec{B} - \vec{\beta} \times \vec{E}) - \frac{\gamma^2}{\gamma + 1}\vec{\beta}(\vec{\beta} \cdot \vec{B})$$

where $\vec{\beta} = \vec{v}/c$ is the velocity three-vector normalized by the speed of light; $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

Problem 5

A conducting rod with mass m slides without friction on two conducting rails in a uniform magnetic field B, which is perpendicular to the plane of motion. See Figure. At time t=0, the rod has a distance L from the vertex V and slides to the right with an instantaneous velocity v_0. The rail and rod have a linear resistivity ρ (resistance per unit length).

1. Calculate the current I through the rod at time t=0.
2. Describe the force exerted on the sliding rod by the magnetic field at time t=0.
3. Calculate the Ohmic power dissipated by the rod and rails at $t = 0$.
4. Calculate the mechanical power done by the magnetic force acting on the rod at $t = 0$ and compare with the Ohmic power.
5. Describe the motion of the rod as a function of time (for $t > t_0$). (You will find a nonlinear differential equation in L, which you are not expected to solve.)