Problem 1

An ideal gas consisting of $N$ particles of mass $m$ (classical statistics being obeyed) is enclosed in an infinitely tall cylindrical container placed in a uniform gravitational field and is in thermal equilibrium. Calculate the classical partition function, Helmholtz free energy, and heat capacity of this system.
Problem 2

Consider an isolated (fixed total energy) system of N atoms, each of which may exist in three states of energies $-\epsilon, 0, +\epsilon$. Let us specify the macrostates of the system by N, E (the total energy) and n (the number of atoms in the zero-energy state).

a) Identify explicitly and write out the microstates corresponding to the N = 3, E = 0, n = 1 and N = 3, E = 0, n = 3 macrostates (use “-,0,+” to denote the state of the atoms).

(2 points)

b) If $n_+$ and $n_-$ are the number of atoms in the $+\epsilon$ and $-\epsilon$ states, show that the macrostate where E=0 one has $n_+ = n_- = (N-n)/2$.

(1 point)

c) Explain why the weight of the macrostate $E = 0$ is

$$\binom{N}{n} \binom{N-n}{(N-n)/2}$$

(2 points)

d) Show that (for large N) the entropy of the $E = 0$ macrostate is given by

$$\frac{S(x)}{Nk_b} = -x \ln(x) - (1 - x) + (1 - x) \ln(2)$$

where $x = n/N$.

(5 points)
Problem 3

Calculate the mean speed, $\langle v \rangle$, and the mean square velocity, $\langle v^2 \rangle$, of the particles of a two-dimensional classical monatomic ideal gas. The gas is composed of particles of mass $m$ and kept at temperature $T$. 