Statistical Mechanics

August 31, 2011

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

An ideal gas consisting of N particles of mass m (classical statistics being obeyed) is enclosed in an infinitely tall cylindrical container placed in a uniform gravitational field and is in thermal equilibrium. Calculate the classical partition function, Helmholtz free energy, and heat capacity of this system.

Problem 2

Consider an isolated (fixed total energy) system of N atoms, each of which may exist in three states of energies $-\epsilon$, 0, $+\epsilon$. Let us specify the macrostates of the system by N, E (the total energy) and n (the number of atoms in the zero-energy state).

a) Identify explicitly and write out the microstates corresponding to the N = 3, E = 0, n =1 and N = 3, E = 0, n =3 macrostates (use "-,0,+" to denote the state of the atoms).
(**2 points**)

b) If n_+ and n_- are the number of atoms in the $+\epsilon$ and $-\epsilon$ states, show that the macrostate where E=0 one has $n_+ = n_- =$ (N-n)/2.
(**1 point**)

c) Explain why the weight of the macrostate $E = 0$ is

$$\binom{N}{n}\binom{N-n}{(N-n)/2}$$

(**2 points**)

d) Show that (for large N) the entropy of the $E = 0$ macrostate is given by

$$\frac{S(x)}{Nk_b} = -x\ln(x) - (1-x) + (1-x)\ln(2)$$

where $x = n/N$.
(**5 points**)

Problem 3

Calculate the mean speed, $\langle v \rangle$, and the mean square velocity, $\langle v^2 \rangle$, of the particles of a *two-dimensional* classical monatomic ideal gas. The gas is composed of particles of mass m and kept at temperature T.