Quantum Mechanics

September 26, 2011

Work 4 (and only 4) of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

1. For a negative charge bound by a heavy positive charge, the following observables are conserved and can be determined simultaneously: energy, total angular momentum, L_z, parity.

   a.) suppose one applies a homogenous **magnetic field** in the $\hat{z}$-direction, which of the above observables are no longer conserved?

   b.) suppose one applies a homogenous **electric field** in the $\hat{z}$-direction, which of the above observables are no longer conserved? (ignore tunneling effects)

   (3 points)

2. Two protons with spin parallel are bound by a harmonic oscillator potential in 3 dimensions

$$H = \frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} + \frac{1}{2}k\left(\vec{r}_1 - \vec{r}_2\right)^2 .$$

   a.) What is the ground state energy? What is the energy spectrum? Note: you must be very specific about the range for any parameters appearing in your answer! Explain how you obtained your answer! (5 points)

   c.) What is the degeneracy of the ground state? Explain how you obtained your answer! (2 points)

Problem 2

A particle is in a three-dimensional box (infinite square well) with all three sides L. If the particle is in the first excited state, what can you tell about its angular momentum?

Problem 3

Consider a bound state solution, $\psi(\mathbf{r})$, of the Schrödinger equation in three dimensions with a spherically symmetric potential. Show that the energy, E is given by

$$E = \frac{\int d\mathbf{r} V(r)\psi(\mathbf{r})}{\int d\mathbf{r}\psi(\mathbf{r})}$$

(note that the integral is over the wave function, not the square of the wave function).
For the case of

$$V(r) = \frac{2\hbar^2}{m}\alpha^2 r^2$$

show explicitly that this relation gives the correct energy by solving for the ground state wave function and energy and comparing with this formula.

Problem 4

Consider a Hamiltonian of the form

$$H = H^0 + H'(t)$$

where $H^0 u_n(x) = E_n u_n(x)$ and H' is to be considered as a perturbation. The full wave function can be expanded in terms of the eigenstates of H^0

$$\psi(x,t) = \sum_m a_m(t) e^{-iE_m t/\hbar} u_m(x)$$

where we will take $< m|m' >= \int u_m^* u_{m'} dx = \delta_{m,m'}$
1) Given the initial conditions $a_n(0) = 1$ and $a_k(0) = 0$ for $k \neq n$ show that.

$$a_k(t) = -\frac{i}{\hbar} \int_0^t < k|H'(t)|m > e^{i\omega_{kn}t'} dt'$$

where $\omega_{kn} = (E_k - E_n)/\hbar$. Assume that (for the range of times being considered) $a_k(t) << 1$ for $k \neq n$
2) If $H'(t) = 2V_0 \cos \omega t$ show that the probability of finding the system in the state f at time t, given that it was in the state i at $t = 0$, is:

$$P_f(t) = |a_f|^2 = \frac{|V_{fi}|^2}{\hbar^2} \left(\frac{\sin^2 \delta}{\delta} \right) t^2$$

where $V_{fi} =< f|V_0|i >$ and $\delta = (\omega_{fi} - \omega)t/2$.
3) If the transition is to a number of states around some energy E_f where the density of states is $N(E_f)$ show that the transition rate is

$$W_{fi} = \frac{P_f(t)}{t} = \frac{2\pi}{\hbar} |V_{fi}|^2 n(\hbar\omega_{fi})$$

This last formula is known as "Fermi's Golden Rule".

Problem 5

a) Set up Schrödinger's equation for the He atom and use the variational method to approximate the energy of the ground state. In this calculation take

$$u = Ae^{-c(r_1+r_2)}$$

for the trial function with c($>$0) as a parameter to be determined by variation.
b) Confirm that your result is, in fact, a minimum.
c) The experimental value for the binding energy ground state of the He atom is -79.0 eV. How does this compare with your answer and how can you explain any discrepancy?