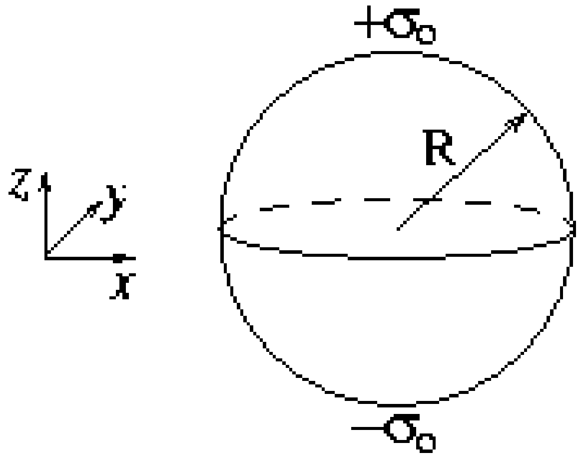
Electromagnetism

August 29, 2011

Work 4 (and only 4) of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A thin spherical shell of radius R is made of a nonconducting material. The upper half of the shell carries a positive uniform surface charge density $+\sigma_0$, while the lower half of the shell carries a negative uniform surface charge density $-\sigma_0$.



(a) Calculate the electric dipole moment of the shell.

(b) Find an approximate electrostatic potential at distances $>> R$.

Problem 2

Note: This problem asks for mathemtical expressions and also for written explanations. Please write neatly and use complete sentences to express your answers. If I cannot read your handwriting or cannot decipher the meaning of a few keywords, I will not give you credit. I prefer SI (MKSA) units.

1. Write down the microscopic Maxwell's equations describing the propagation of plane waves in vacuum (empty space).
2. Explain the meaning of the various terms and the significance of the four equations. (I am looking for 1-3 sentences for each of the four Maxwell's equations.)
3. Which terms in the general microscopic Maxwell's equations are missing, because they have no significance in empty space (vaccum).
4. Show that the plane wave
$$\vec{E}\left(\vec{r},t\right) \;=\; \vec{E}_0 \exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right].$$
is a solution to Maxwell's equations. How do we need to choose the magnetic field belonging to this wave?
5. What are the relationships between the wave vector $\vec{k}$ and the electric and magnetic fields?
6. Which of Maxwell's equations are related to the transverse character of an electromagnetic wave?
7. Derive the wave equation from Maxwell's equations in vaccum.
8. Derive the dispersion relation (i.e., the relationship between the wave vector and angular frequency) for an electromagnetic field in vaccum? Explain your answer!
9. How do we need to modify Maxwell's equations to describe the propagation of an electromagnetic field in an ideal homogeneous dielectric? Write down the modified (macroscopic) Maxwell's equations. Explain the differences and any new terms.

Hint: From the inside cover of Jackson:
$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{a}\right) \;=\; \vec{\nabla}\left(\vec{\nabla}\cdot\vec{a}\right)-\vec{\nabla}^2\vec{a}.$$
You will also need
$$\vec{\nabla}\times\vec{E}_0\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right] \;=\; i\vec{k}\times\vec{E}_0\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right].$$

Problem 3

a) Show that the energy of a quadrupole in an external electric field E,

$$U_{quad} = -\frac{1}{6} Q_{ij} \frac{\partial E_j}{\partial xi}$$

in terms of its quadrupole tensor Q_{ij}, can be rewritten as

$$U_{quad} = -\frac{1}{4} Q_{xx} \frac{\partial E_x}{\partial x}$$

if the quadrupole is rotationally symmetric about the x axis. Give an expression for the force F on the quadrupole. (**4 points**).

b) A rotationally symmetric quadrupole of strength Q_{xx} (zero net charge, zero dipole moment) is located at a distance r from a point charge q. What is the force on the quadrupole if: i. The x axis is along the line joining Q_{xx} and q? ii. The x axis is perpendicular to the line joining Q_{xx} and q? (**6 points**)

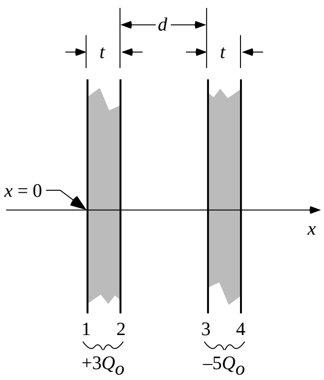
Problem 4

X-rays are widely used to gain insight into the structure of materials through diffraction experiments. In this problem we will investigate the interaction between x-rays and a metallic surface.

a) [**5 points**] X-rays which strike a metal surface at an angle of incidence with the surface normal that is greater than a critical angle (θ_0) are totally reflected. Assuming that a metal contains n free electrons per unit volume, determine θ_0 as a function of the angular frequency (ω) of the incident x-rays.

b) [**5 points**] For the case that ω and θ are such that total reflection does not occur, derive a mathematical expression for the fraction of the incident light that is reflected? You may assume for simplicity that the polarization vector of the x-rays is perpendicular to the plane of incidence.

Problem 5



A region near the center of a very large parallel-plate capacitor is shown. Each conducting plate has area A and thickness t. There is an overall charge $+3Q_0$ on the left plate; the right plate is has an overall charge that is opposite in sign $-5Q_0$. There is a distance d between the plates.

(a) Give an expression for the electric field for all points along the x-axis shown.

(b) Find the charge density on each of the numbered surfaces 1-4.

(c) Starting with the definition of capacitance $C = Q/V$, show that the capacitance is $C = \epsilon_0 A/d$ as expected.

Consider a spherical capacitor consisting of two concentric conducting shells, each of thickness t. The distance from the center to the outer surface of the inner shell (Surface 2) is R, and the distance between the two conductors is d. There is an overall charge $+3Q_0$ on the inner shell; the outer shell has an overall charge that is opposite in sign $-5Q_0$.

(d) Give an expression for the electric field as a function of r, the distance from the origin.

(e) Find the charge density on each of the numbered surfaces 1-4.

(f) Find an expression for the capacitance. Show that as R becomes very large compared to d, the capacitance of the shells is approximately $\epsilon_0 A/d$.