Classical Mechanics

August 31, 2011

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

A half-disk of radius $R$ and mass density $\rho$ (mass per unit area) can roll without slipping on flat surface in the $xy$-plane.

The total mass of the disk is

$$m = \int_{\text{disk}} \rho \, dx \, dy.$$ 

In the diagram the disk is shown in its equilibrium position. What is the frequency of small rolling oscillations of the disk with respect to equilibrium?
Problem 2

Derive Snell’s Law of refraction
Problem 3

Assume that the Lagrangian for a system performing 1-dimensional motion is given by

\[ L = e^{\gamma t} \left( \frac{1}{2} mq^2 - \frac{1}{2} kq^2 \right) \]

where \( \gamma, m, \) and \( k \) are positive constants.

a) [2 points] Are there any constants of motion?
b) [3 points] Determine \( q(t) \) and describe the motion? Describe all cases.
c) [3 points] Suppose a point transformation is made to another generalized coordinate \( S \) given by

\[ S = e^{\gamma t/2} q \]

What is the Lagrangian? What is the Lagrange equation? Are there any constants of motion? If so, derive their mathematical form.
d) [2 points] Describe the relationship between the two solutions. Especially comment on your findings for the constants of motion and how you reconcile any differences that you may find.