Classical Mechanics
September 22, 2010

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

A massless slider moves without friction along the x-axis. A pendulum of length $l$ and mass $m$ hangs from the slider. In your answer, use the coordinate $x$ of the slider and the angle $\theta$ of the pendulum, defined in the diagram below.

Is there a possible motion for this system where the coordinate $\theta$ has a constant non-zero value? That is, is it possible for the slider+pendulum to move in such a way that the angle of the pendulum is unchanging?

Then,

if the answer is **no**, describe a slight modification to the problem which would make this motion possible.

if the answer is **yes**, determine the initial conditions (the values of $x$, $\dot{x}$, $\theta$, $\dot{\theta}$ at $t = 0$) that could produce this motion.

Be sure to carefully explain your answer in either case.
Problem 2

A plank of length \( L \) and mass \( M \), which can be treated as arbitrarily thin for the purposes of this problem, has one end on a smooth horizontal floor and the other end against a smooth vertical wall, such that the plane in which it is located (and remains located throughout its motion) is perpendicular to the wall. The plank is initially at rest, forming an angle \( \theta_0 \) with the floor.

a. Make a convenient choice of generalized coordinates and find the Lagrangian.

b. Derive the corresponding equations of motion.

c. Prove that the plank leaves the wall when its upper end has fallen to a height \((2/3)L \sin \theta_0\).
Problem 3

a) Set up and derive the solution for damped harmonic motion.
b) Set up and derive the solution for a damped harmonic oscillator with a sinusoidal forcing function.