

Statistical Mechanics

September 2, 2009

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

Consider a simple mono-atomic solid consisting of N atoms. Assume that the Einstein approximation can be used to describe the vibrational motion of the atoms.

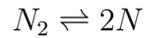
- a. Assume that the vibrational level spacing has a value of $\epsilon = 0.037$ eV. Consider the probability that an atom of this solid is in the ground state. At what temperature is this probability equal to 0.1?
- b. For a given N and ϵ , how high does the temperature need to be to ensure that effectively no atoms are in the ground state? Is such temperature achievable in an ordinary solid?
- c. How would hydrostatic pressure affect the above observations?

Problem 2

Evaluate the equilibrium constant

$$K_P = \frac{P_N^2}{P_{N_2}}$$

of the dissociative reaction



at a temperature of 5000 K under the following assumptions:

1. The characteristic temperatures of rotation and of vibration of the N_2 molecule are

$$\begin{aligned}\Theta_r &= 2.84\text{ K} \\ \Theta_v &= 3.36 \times 10^3\text{ K} ,\end{aligned}$$

respectively.

2. The dissociation energy is $D_0 = 169.3\text{ kcal/mole}$. This includes the correction for the zero point energy of the vibrations.
3. The electronic ground state of the N_2 molecule has no degeneracy but the electronic ground state of the N atom has a degeneracy of 4 due to electron spin.

Problem 3

Consider an ideal gas composed of N absolutely rigid (i.e. no vibrations) diatomic molecules. Each molecule consists of two atoms of masses m and M , separated by distance d . The gas is held at constant volume V and temperature T . **(a)** Find the partition function of a single molecule, assuming that $T \gg \hbar^2(M + m)/Mmd^2k_B$. **(b)** Starting from the total partition function, calculate the internal energy and specific heat of this gas at high temperature. Show that your result is consistent with the equipartition theorem.

