Electromagnetism
February 5, 2007

Work 4 of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

The electrostatic potential on the surface of a sphere of radius $R$ is given by

$$V(\theta, \phi) = V_0 \sin^2 \theta.$$ 

Solve the boundary-value problem for the sphere and find the electrostatic potential everywhere outside the sphere.

*Hint*: Recall that $P_0(\cos \theta) = 1$, $P_1(\cos \theta) = \cos \theta$, $P_2(\cos \theta) = \frac{3}{2} \cos^2 \theta - \frac{1}{2}$. 
Problem 2

Consider two molecules; molecule 1 acts as an emitter and molecule 2 acts as a receiver. Consider molecule 1 as a classical dipole oscillator, oscillating with a frequency $\omega = 2\pi \nu$ and an amplitude $A_0$, so that the displacement is $A = A_0 \cos \omega t$. The charge of the oscillator (molecule 1) is $q$. Molecule 2 is a distance $r$ away from molecule 1. The molecules are immersed in a medium with a refractive index $n$.

Show that the electrical field at the site of molecule 2 has the components

$$E_r = 2qA_0 \cos \theta \left[ \frac{1}{n^2 r^3} \cos \left( \omega \left\{ t - \frac{rn}{c} \right\} \right) - \frac{\omega}{ncr^2} \sin \left( \omega \left\{ t - \frac{rn}{c} \right\} \right) \right]$$

for the field along $r$, and

$$E_t = qA_0 \sin \theta \left[ \left( \frac{1}{n^2 r^3} - \frac{\omega^2}{c^2 r^2} \right) \cos \left( \omega \left\{ t - \frac{rn}{c} \right\} \right) - \frac{\omega}{ncr^2} \sin \left( \omega \left\{ t - \frac{rn}{c} \right\} \right) \right]$$

for the field perpendicular to $r$. 
Problem 3

Answer all 5 parts and use diagrams where appropriate:

(A) What is the difference between electrical and thermal conductance?

(B) How do we measure the electrical conductivity of a material (draw a picture and write down the equation)?

(C) What is the difference between surface and bulk conductivity?

(D) Write down the equation for a diode which describes the current-voltage characteristics, and explain the role of the diode quality factor $n$.

(E) Explain what is meant by forward and reverse bias (draw diagrams of a $PN$ or $Schottky$ diode to explain).
Problem 4

If a particle of mass $m$ and charge $q$ is dropped from rest from a height $h$ above the surface of the Earth and falls through a uniform magnetic field $B$ directed parallel to the Earth’s surface, neglecting atmospheric drag effects:

(A) Write down the equations of motion for the particle in terms of the cyclotron frequency $\omega = qB/m$.

(B) Solve these equations for the velocity and position of the particle as a function of time.

(C) Show that, if $\omega$ exceeds a certain value, the particle does not strike the ground.

(D) Sketch the particle trajectory in this case.
Problem 5

Derive an expression for the vector potential, $A$, produced by an infinitely long solenoid. (the pitch of the winding may be neglected). Give the result for both the region outside and within the solenoid.