Statistical Mechanics

September 12, 2007

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

The understanding of the thermodynamic properties of radiation has played a fundamental role in physics. We are reminded of milestones in physics such as Wien's displacement law, the Rayleigh-Jeans law, the Stefan-Boltzmann law and black-body radiation. In this problem you will use thermodynamics and quantum statistics to treat a 3-dimensional photon gas with energy spectrum $E = \hbar cq$, where c is the speed of light and $q = |\vec{q}|$ is the modulus of the wave vector $\vec{q}$.

a. Explain why the chemical potential of a photon gas is zero. A concise qualitative argument is sufficient.

b. Use quantum statistics to show that the pressure of a 3-dimensional photon gas is given by

$$P \propto T^4$$

where T is the absolute temperature. (Hint: The photon gas is confined to a large volume $V = L^3$.)

c. Continuing the argument in b. show that the relationship between internal energy density ($u = U/V$, where U is the internal energy, and V is the volume) and pressure for a 3-dimensional photon gas is given by

$$u = 3P$$

d. Using the 2$^{\text{nd}}$ law of thermodynamics, prove the following relation:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

and show using the result from c. that the same temperature dependence of pressure results as obtained in b.

Problem 2

Consider an ideal gas consisting of N particles obeying classical statistics. Suppose that the energy of one particle ϵ is proportional to the magnitude of momentum p, $\epsilon = cp$. Find the thermodynamic functions (Helmholtz free energy, F; pressure, P; internal energy, U; enthalpy, H; Gibbs free energy, G; specific heats c_V and c_P) of this ideal gas without considering the internal structure of the particles.

Problem 3

A classical monatomic ideal gas in thermal equilibrium is enclosed in a vertical cylinder of height h and placed in a uniform gravitational field g. The gas is composed of identical particles of mass m. Calculate the average *potential* energy of a gas particle. What is the average potential energy of a particle in cases when (i) the cylinder is infinitely long ($h >> kT/mg$) and (ii) the cylinder is short ($h << kT/mg$)?