Electromagnetism

September 10, 2007

Work 4 of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

The electronic cloud in the hydrogen atom amounts to an average volume charge density

$$\rho(r) = -\frac{e}{\pi a_0^3} e^{\frac{-2r}{a_0}}$$

where e is the magnitude of the fundamental charge of the electron, a_0 is the Bohr radius and r is the distance from the proton's position.

(a) Verify that the dimensions of ρ are correct.

(b) Verify that the total charge of the electronic cloud is $-e$.

(c) Write down an explicit expression for the (point-like) charge density of the proton as well as an expression for the total charge density.
HINT: The point-like proton has a density proportional to a notoriously singular function.

(d) Find the resultant electric field, $\mathbf{E}$, in the hydrogen atom.

(e) Investigate the electric field $\mathbf{E}$ for the two limiting cases: $r >> a_0$ and $r << a_0$.
Explain the results of your findings.

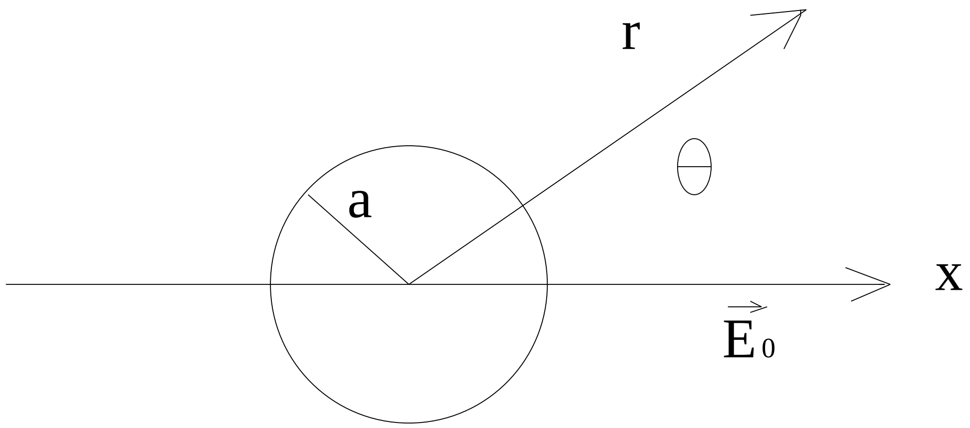
(f) Sketch the graph for the magnitude of the electric field versus radial distance.

Problem 2

An uncharged conducting sphere of radius, a, in vacuum is placed in a uniform electric field, E_0, directed along the x axis (see figure). Assume the potential is given by

$$Ar\cos\theta + B\cos\theta/r^2,$$

where A and B are constants to be determined by the boundary conditions.
Determine:
(a) the constants A and B,
(b) the potential at any point inside the sphere,
(c) the charge/unit area at any point on the surface of the sphere.



Problem 3

Estimate the penetration depth of an infrared electromagnetic wave with a wavelength of $\lambda = 1500\text{nm}$ in pure water. Assume that the wave is normally incident on the surface of water and the complex permittivity of pure water at this wavelength is $\epsilon \approx 1.7(1 + 0.001i)\epsilon_0$.

Hint: The penetration depth is defined as the distance at which the field amplitude is reduced to 1/e of its original value.

Problem 4

$SmCo_5$ is one of the strongest permanent-magnet materials. Consider two equal $SmCo_5$ spheres with a radius R=1 cm. and a magnetization $M = 7.5 \times 10^5$ J/Tm^3. The spheres are stuck together with unlike poles touching. What force (in Newtons) must be applied to separate them?

Problem 5

A popular physics demonstration consists of a strong magnet falling through a conducting tube. A first step in analyzing these effects is to understand the motion of a magnet falling through a thin conducting ring. We'll model the magnet as a pure dipole. The vector potential for a magnetic dipole is

$$\mathbf{A}_{dipole} = \frac{\mu_0}{4\pi}\frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}.$$

(a) Find an expression for the magnetic field for the dipole, assuming that the north pole is at the top of the magnet.
At the instant that the center of the magnet is at the center of the ring, and moving with a velocity v:
(b) Find the magnetic flux through the conducting ring.
(c) Find the emf in the loop.
At the instant that the center of the magnet is at a height h above the center of the ring, and moving with a velocity v':
(d) Find the magnetic flux through the conducting ring.
(e) At this instant in time, what is the direction of the current? Explain how you know.
(f) The most expensive part of this demonstration is the thick-walled copper tube. Explain why a thinner-walled tube would not be as effective.