Electromagnetism

September 12, 2005

Work 4 of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

(Nakotte)
In electro*statics*, the Maxwell stress tensor is given by

$$T_{\alpha\beta} = \frac{1}{4\pi}(E_\alpha E_\beta - E^2\delta_{\alpha\beta}/2) \ .$$

a.) Show that

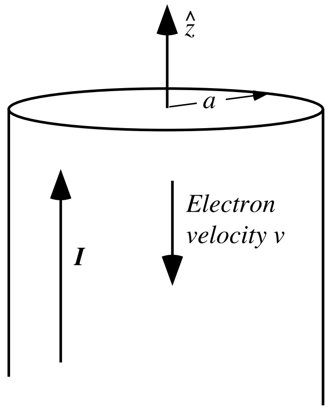
$$\int_{\partial V} T_{\alpha\beta} ds_\alpha = \int_V \rho E_\beta dV \ ,$$

where V is a volume with boundary ∂V, $d\vec{s}$ is the infinitesimal surface element on the boundary, ρ is the charge density and $\vec{E}$ is the electric field.

b.) Use this result to calculate the net force which the two hemispheres of a uniformly charged solid sphere with radius R and total charge Q exert on each other.

Problem 2

(Kanim)
Consider a solid copper wire of radius a conducting a current I. Assume that the wire has a conduction electron velocity v that is uniform throughout the volume of the wire. Let the lattice volume charge density ρ^+ (i.e., the charge density due to all charges *except* the conduction electrons) also be constant throughout the wire. In the reference frame of the lattice:



a.) Find the radial electric field $\vec{E}^+(r)$ inside the wire due to the lattice alone.

b.) Find an expression for the radial electric field $\vec{E}^-(r)$ inside the wire due to the conduction electrons alone, in terms of an integral over the conduction electron charge density $\rho^-(r, v)$.

c.) Find an analogous expression for the radial magnetic field $\vec{B}^-(r)$ inside the wire due to the conduction electrons.

d.) Show that $\rho^-(r, v)$ is in fact independent of r under the given conditions.

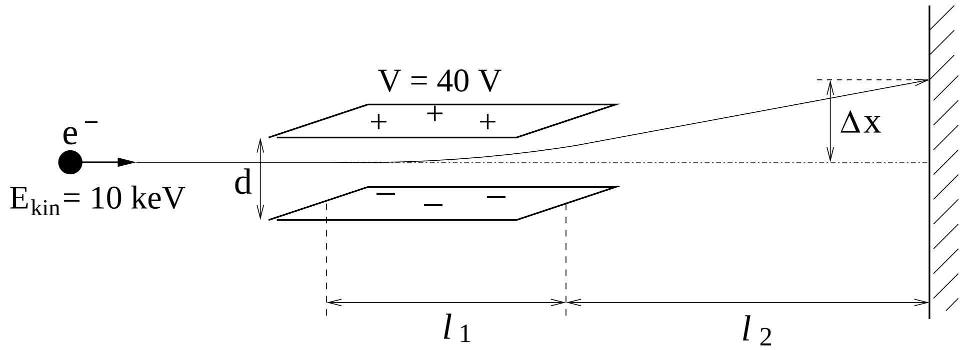
e.) Find $\rho^-(v)$ in terms of ρ^+.

f.) Assume the wire is neutral overall. What is the surface charge density of the wire?

g.) Estimate this surface charge density for a household wire with a diameter of $2\,mm$ and a current of $5\,A$.

Problem 3

(Vasiliev)
An electron with the kinetic energy $E_{kin} = 10$ keV flies through a parallel plate capacitor. The potential difference between the plates of the capacitor is $V = 40$ V. The length of the capacitor is $l_1 = 10$ cm and the distance between the plates is $d = 1$ cm. Calculate the lateral displacement of the electron, Δx, on a screen located at a distance $l_2 = 20$ cm from the capacitor.



Problem 4

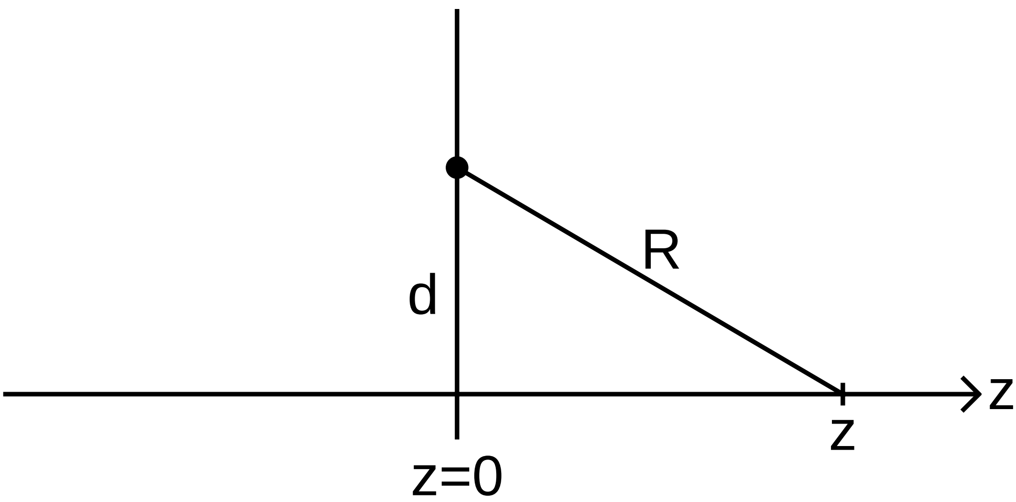
(Armstrong)
The electric and magnetic fields a distance d from an infinite wire carrying a current

$$I = 0 \ (t \leq 0), \ \ I = \alpha t \ (t > 0)$$

may be described by scalar and vector potentials at point P.

$$V = 0, \quad \mathbf{A} = \frac{\mu_0 \hat{\mathbf{k}}}{4\pi} \int dz \frac{I(t_r)}{R}$$

where t_r is the retarded time, $R = \sqrt{z^2 + d^2}$ and $\hat{\mathbf{k}}$ is a fixed unit vector along the z axis.



a) From your knowledge of currents in wires, justify setting $V = 0$ above.
b) Explain why $\mathbf{A}$ remains zero for some initial time $t_0 > 0$ after the current in the wire begins (what is t_0?).
c) For a given time $t > 0$, only a finite length of the wire, $-z_{max} \leq z \leq +z_{max}$, is responsible for $\mathbf{A}$. What is z_{max}
d) For $t > t_0$ compute $\mathbf{A}$.
e) You don't need to compute anything here, but simply state how, knowing $\mathbf{A}$ and V, you would compute $\mathbf{E}$ and $\mathbf{B}$ (be as quantitative as you can).

Problem 5

(Urquidi)
The circuital form of the Biot-Savart law is

$$\vec{B}(r_t) = \frac{\mu_0 I_s}{4\pi} \oint_s \frac{d\vec{r}_s \times \hat{R}_{st}}{R^2} \tag{1}$$

It is possible to obtain from this form a simple, yet very useful result, namely:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I \tag{2}$$

From this we can use what we know about magnetostatics to find that $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$. If $\vec{J}$ is known everywhere, we then know $\vec{\nabla} \times \vec{B}$ everywhere. This is half the knowledge required by the Helmholtz theorem for the determination of $\vec{B}$ itself. Derive (2) using (1) as your starting point.