Problem 1

(Nakotte)
In electrostatics, the Maxwell stress tensor is given by

\[ T_{\alpha \beta} = \frac{1}{4\pi} (E_\alpha E_\beta - E^2 \delta_{\alpha \beta}/2) . \]

a.) Show that

\[ \int_{\partial V} T_{\alpha \beta} d\mathbf{s}_\alpha = \int_V \rho E_\beta dV , \]

where \( V \) is a volume with boundary \( \partial V \), \( d\mathbf{s} \) is the infinitesimal surface element on the boundary, \( \rho \) is the charge density and \( \mathbf{E} \) is the electric field.

b.) Use this result to calculate the net force which the two hemispheres of a uniformly charged solid sphere with radius \( R \) and total charge \( Q \) exert on each other.
Problem 2

(Kanim)
Consider a solid copper wire of radius $a$ conducting a current $I$. Assume that the wire has a conduction electron velocity $v$ that is uniform throughout the volume of the wire. Let the lattice volume charge density $\rho^+$ (i.e., the charge density due to all charges except the conduction electrons) also be constant throughout the wire. In the reference frame of the lattice:

a.) Find the radial electric field $\vec{E}^+(r)$ inside the wire due to the lattice alone.

b.) Find an expression for the radial electric field $\vec{E}^-(r)$ inside the wire due to the conduction electrons alone, in terms of an integral over the conduction electron charge density $\rho^-(r, v)$.

c.) Find an analogous expression for the radial magnetic field $\vec{B}^-(r)$ inside the wire due to the conduction electrons.

d.) Show that $\rho^-(r, v)$ is in fact independent of $r$ under the given conditions.

e.) Find $\rho^-(v)$ in terms of $\rho^+$.

f.) Assume the wire is neutral overall. What is the surface charge density of the wire?

g.) Estimate this surface charge density for a household wire with a diameter of 2 mm and a current of 5 A.
Problem 3

(Vasiliev)

An electron with the kinetic energy $E_{\text{kin}} = 10 \text{ keV}$ flies through a parallel plate capacitor. The potential difference between the plates of the capacitor is $V = 40 \text{ V}$. The length of the capacitor is $l_1 = 10 \text{ cm}$ and the distance between the plates is $d = 1 \text{ cm}$. Calculate the lateral displacement of the electron, $\Delta x$, on a screen located at a distance $l_2 = 20 \text{ cm}$ from the capacitor.
Problem 4

(Armstrong)
The electric and magnetic fields a distance $d$ from an infinite wire carrying a current

$$I = 0 \ (t \leq 0), \ I = \alpha t \ (t > 0)$$

may be described by scalar and vector potentials at point P.

$$V = 0, \quad A = \frac{\mu_0 \hat{k}}{4\pi} \int dz \frac{I(t_r)}{R}$$

where $t_r$ is the retarded time, $R = \sqrt{z^2 + d^2}$ and $\hat{k}$ is a fixed unit vector along the $z$ axis.

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a) From your knowledge of currents in wires, justify setting $V = 0$ above.
b) Explain why $A$ remains zero for some initial time $t_0 > 0$ after the current in the wire begins (what is $t_0$?).
c) For a given time $t > 0$, only a finite length of the wire, $-z_{max} \leq z \leq +z_{max}$, is responsible for $A$. What is $z_{max}$
d) For $t > t_0$ compute $A$.
e) You don’t need to compute anything here, but simply state how, knowing $A$ and $V$, you would compute $E$ and $B$ (be as quantitative as you can).
Problem 5

(Urquidi)
The circuital form of the Biot-Savart law is

\[ \vec{B}(r_t) = \frac{\mu_0 I_s}{4\pi} \oint_s \frac{d\vec{r}_s \times \hat{R}_{st}}{R^2} \]  

(1)

It is possible to obtain from this form a simple, yet very useful result, namely:

\[ \oint \vec{B} \cdot d\vec{r} = \mu_0 I \]  

(2)

From this we can use what we know about magnetostatics to find that \( \nabla \times \vec{B} = \mu_0 \vec{J} \). If \( \vec{J} \) is known everywhere, we then know \( \nabla \times \vec{B} \) everywhere. This is half the knowledge required by the Helmholtz theorem for the determination of \( \vec{B} \) itself. Derive (2) using (1) as your starting point.