Geophysics

January 28, 2005

Work 4 of the 5 problems. Put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A simple seismometer is a damped harmonic pendulum.

a) Set up the differential equation of motion of the pendulum bob for an input ground motion.

b) Solve the equation of motion to get the response of the seismometer bob for an input motion. (I recommend Laplace or Fourier transforms so that the final time-domain response can be expressed as a convolution with the ground motion input).

Problem 2

Consider a 1-D earthquake location problem with the only two parameters being distance and origin time. The data are:

$$x = 0\ km,\ t = 1.45\ s$$

$$x = 3\ km,\ t = 1.20\ s$$

$$x = 7\ km,\ t = 1.35\ s$$

$$x = 10\ km,\ t = 1.5\ s$$

Note that the inversion for location and time is a non-linear inverse problem because the travel-time depends only on the absolute value of the relative event-receiver distance.

a) Linearize the travel-time equation and locate the event. As a starting position use x=5 km and t=0.0 s. Do the first iteration but just set up the second. (Hint: the derivative of abs$(x - x_0)$ is sign$(x - x_0)$).

b) Make an estimate of standard error of the data followed by estimates of the standard errors for the model parameters.

Problem 3

The following wave amplitudes are measured:

$$x = 10\ km,\ \ A = 82$$

$$x = 100\ km,\ \ A = 21$$

We wish to fit these to an attenuation curve of form

$$A(x) = \frac{Ae^{-ax}}{x^b}$$

where A is the initial amplitude at $x = 0$, a is the spatial decay coefficient, and b is the geometric spreading factor.

a) Linearize this equation by taking an appropriate transform. (We do not want an iterative solution.)

b) Set up a damped-least-squares solution to estimate the three unknowns A, a, and b, but do not invert the 3x3 matrix.

c) In terms of the matrix of part b, set up the data resolution matrix. Again, you do not need to carry out the matrix inversions.

Problem 4

Consider a 30 km crust of velocity 6.5 km/s over a mantle of velocity 8.0 km/s.

a) For a surface point source, plot a travel-time curve (time vs. distance) for this model. Be sure to include the direct, refracted, and reflected phases. On the plot label the intercepts and slopes of all lines drawn and identify them.

b) If we now allow the Moho to dip 5 degrees downward, what is the apparent velocity of the wavefront measured at the surface?

Problem 5

Consider a plane SH wave incident on a welded solid-solid planar boundary.

a) What are the boundary conditions needed to find the reflection and transmission coefficients?

b) Find the reflection and transmission coefficients.